Kvaterniomuistilista

Perusratkaisut

$$\text{Helmholz}\quad \triangle + \alpha^2 I: \quad \phi_\alpha(x) = -\frac{e^{i\alpha|x|}}{4\pi|x|}$$

$$\text{Dirac}\quad \not{\partial}_\alpha: \quad K_\alpha(x) = -\not{\partial}_{-\alpha}\phi_\alpha(x) = (\alpha + \frac{x}{|x|^2} - i\alpha\frac{x}{|x|})\phi_\alpha(x)$$

Kerrospotentiaalit ja muita operaattoreita:

$$T_\alpha f(x) = \int_\Omega K_\alpha(x-y)f(y)\mathrm{d}y, \qquad x \in \mathbb{R}^3$$

$$K_\alpha f(x) = -\int_{\partial\Omega} K_\alpha(x-y)\nu(y)f(y)\mathrm{d}s(y), \qquad x \notin \Omega$$

$$S_\alpha f(x) = -2\int_{\partial\Omega} K_\alpha(x-y)\nu(y)f(y)\mathrm{d}s(y), \qquad x \in \partial\Omega$$

$$P_\alpha = \frac{1}{2}(I + S_\alpha)$$

$$Q_\alpha = \frac{1}{2}(I - S_\alpha)$$

Lauseita

Borel–Pompeiu: $f = (K_\alpha + T_\alpha\not{\partial}_\alpha)f$ sisäalueessa.
Cauchyn kaava sisäalueessa: $f = K_\alpha f$ sisäalueessa, kun $\not{\partial}_\alpha f = 0$.
Cauchyn kaava ulkoalueessa: $f = -K_\alpha f$ ulkoalueessa, kun $\not{\partial}_\alpha f = 0$ ja f toteuttaa säteilyehdon (6.4)

Säteilyehtoja:

Olkoon

$$u_\pm(x) := -\frac{e^{ik|x|}}{4\pi|x|}$$

$$K^\pm(x) := -\not{\partial}_{\mp k}u^\pm(x)$$

Sommerfeldin säteilyehdot:
$(\partial_r \mp ik)u_\pm(x) = o(1/|x|)$, kun $r = |x| \to \infty$.
Säteilyehto (6.3):
$(\alpha - \frac{x}{|x|^2} + i\alpha\frac{x}{|x|})K(x) = o(1/|x|)$, kun $|x| \to \infty$.
Säteilyehto (6.4):
$(1 + i\frac{x}{|x|})f(x) = o(1/|x|)$, kun $|x| \to \infty$.
Säteilyehto (6.4'):
$(1 - i\frac{x}{|x|})f(x) = o(1/|x|)$, kun $|x| \to \infty$.

Plemeljin lause

Liapunov-alueessa Ω Hölder-jatkuvalla f reunalla

$$(K_\alpha^\pm f)(\tau) = \lim_{x\in\Omega^\pm, x\to\tau}(K_\alpha f)(x) = \pm\frac{1}{2}(I \pm S_\alpha)f(\tau)$$

jokaisella $\tau \in \partial\Omega$.

Operaattori $\not{\partial} + M^\alpha$

$$M_\alpha f := f\alpha,\ \alpha \in \mathbb{H}(\mathbb{C}),$$

$$\lambda := \sqrt{\alpha_v^2},\ \mathrm{Im}\lambda \geq 0$$

$$\xi_\pm := \alpha_0 \pm \lambda,$$

$$P^\pm := \frac{1}{2\lambda}M^{\lambda\pm\alpha_v},$$

$$\triangle W = \delta, \quad \text{eli}$$

$$(Wf)(x) = \int_{\mathbb{R}^3}\phi_0(x-y)f(y)dy, \qquad f \in C_0^\infty(\mathbb{R}^3)$$

Operaattori T_α, kun $\alpha \in \mathbb{H}(\mathbb{C})$

$$T_\alpha = \begin{cases} P^+T_{\xi_+} + P^-T_{\xi_-}, & \alpha \notin \mathcal{G}, \alpha_v^2 \neq 0, \\ (I + M^{\alpha_v}\frac{\partial}{\partial\alpha_0})T_{\alpha_0}, & \alpha \notin \mathcal{G}, \alpha_v^2 = 0, \\ P^+T_{2\alpha_0} + P^-T_0, & \alpha \in \mathcal{G}, \alpha_0 \neq 0, \\ T_0 + M^\alpha W, & \alpha \in \mathcal{G}, \alpha_0 \neq 0, \end{cases}$$

Operaattorit K_α ja S_α, kun $\alpha \in \mathbb{H}(\mathbb{C})$

$$(Vf)(x) := \int_{\partial\Omega}\phi_0(x-y)\nu(y)f(y)\mathrm{d}s(y), \qquad x \notin \partial\Omega$$

$$(\widehat{V}f)(x) := 2\int_{\partial\Omega}\phi_0(x-y)\nu(y)f(y)\mathrm{d}s(y), \qquad x \in \partial\Omega$$

$$K_\alpha = \begin{cases} P^+K_{\xi_+} + P^-K_{\xi_-}, & \alpha \notin \mathcal{G}, \alpha_v^2 \neq 0, \\ (I + M^{\alpha_v}\frac{\partial}{\partial\alpha_0})K_{\alpha_0}, & \alpha \notin \mathcal{G}, \alpha_v^2 = 0, \\ P^+K_{2\alpha_0} + P^-K_0, & \alpha \in \mathcal{G}, \alpha_0 \neq 0, \\ K_0 - M^\alpha V, & \alpha \in \mathcal{G}, \alpha_0 \neq 0, \end{cases}$$

$$S_\alpha = \begin{cases} P^+S_{\xi_+} + P^-S_{\xi_-}, & \alpha \notin \mathcal{G}, \alpha_v^2 \neq 0, \\ (I + M^{\alpha_v}\frac{\partial}{\partial\alpha_0})S_{\alpha_0}, & \alpha \notin \mathcal{G}, \alpha_v^2 = 0, \\ P^+S_{2\alpha_0} + P^-S_0, & \alpha \in \mathcal{G}, \alpha_0 \neq 0, \\ S_0 - M^\alpha \widehat{V}, & \alpha \in \mathcal{G}, \alpha_0 \neq 0, \end{cases}$$

Lauseita

Merkitään $\not{\partial}_\alpha := \not{\partial} + M^\alpha$.
Borel–Pompeiu: $f = (K_\alpha + T_\alpha\not{\partial}_\alpha)f$ sisäalueessa.
Cauchyn kaava sisäalueessa:
$f = K_\alpha f$ sisäalueessa, kun $\not{\partial}_\alpha f = 0$.
T_α oikeanpuoleinen käänteiskuvaus:
$f = \not{\partial}_\alpha T_\alpha f$ sisäalueessa.

Säteilyehdot, kun $\alpha \in \mathbb{H}(\mathbb{C})$

$\alpha \notin \mathcal{G}$, $\alpha_v^2 \neq 0$ ja $\alpha_0 \in \mathbb{R}$:

$$(1 + \tfrac{ix}{|x|})P^+f(x) + (1 - \tfrac{ix}{|x|})P^-f(x) = o(1/|x|)$$

$\alpha \notin \mathcal{G}$, $\alpha_v^2 = 0$:

$$(1 + \tfrac{ix}{|x|})f(x) = o(1/|x|), \qquad \text{ja}$$

$$(1 + \tfrac{ix}{|x|})f(x)\alpha_v = o(1/|x|^2),$$

$\alpha \in \mathcal{G}$, $\alpha_0 \neq 0$:

$$(1 + \tfrac{ix}{|x|})P^+f(x) = o(1/|x|), \qquad \text{ja}$$

$$P^-f(x) = o(1),$$

$\alpha \in \mathcal{G}$, $\alpha_0 = 0$:

$$f(x) = o(1), \qquad \text{ja}$$

$$f(x)\alpha = o(1/|x|),$$