Mathematical Finance , Middle Term Exam , (21.10.08),

Huom. You can use a pocket calculator if you want, but it is not allowed to use books or lecture notes. The duration of the exam is 4 hours. The next lectures of this course will be on monday 27.10.

Part I

In a discrete probability space $\Omega = \{\omega_1, \omega_2, \omega_3\}$, with $P(\{\omega_1\}) = 0.6$, $P(\{\omega_2\}) = 0.1$, $P(\{\omega_3\}) = 0.3$, the random variables $S^0(\omega), S^1(\omega)$ are stock prices at time $t = 1$ in the one-period market model. Note that we do not have the possibility to use any other instruments, for example you cannot use another bank account with deterministic return.

The prices of S^0 and S^1 at time $t = 0$ are $\pi^0 = \pi^1 = 1$, respectively. At time $t = 1$ the random variables take values

$$S^0(\omega_1) = 1.8, \quad S^0(\omega_2) = S^0(\omega_3) = 0.8,$$
$$S^1(\omega_1) = 0.6, \quad S^1(\omega_2) = 1.6, \quad S^1(\omega_3) = 0.8.$$

1) Using S_0 as a numeraire. find all equivalent risk-neutral measures $Q \sim P$, and answer the questions:

Is the market free of arbitrage ? Is the market complete ?

2) Find also all equivalent risk-neutral measures $Q \sim P$ with respect to the numeraire S_1.

In the previous market model, let $X(\omega) := \mathbf{1}(\omega = \omega_1)$ be the a digital option which at time $t = 1$ takes value 1 if $\omega = \omega_1$, 0 otherwise.

3) Compute the set of arbitrage free prices for the digital option $X(\omega)$.

Let's check now the extended stock market where we assign at time $t = 0$ the inital price $c(X) = 0.2$ to the digital-option $X(\omega)$.

4) Show that the extended market model $(S^0, S^1, X; \pi^0, \pi^1, c(X))$ is arbitrage free and complete.

5) Compute the price and hedging strategy of another digital option $Y(\omega) := \mathbf{1}(\omega = \omega_2)$ in the extended market model $(S^0, S^1, X; \pi^0, \pi^1, c(X))$.