Graphical Models
Part Two

Joint probability distribution

- Product of factors
- $P(\alpha, \beta, \tau, Y) = N(\alpha| 0, 1.0E\text{-}3)\ N(\beta| 0, 1.0E\text{-}3)\ \text{Gamma}(\tau| 1.0E\text{-}3, 1.0E\text{-}3) \prod N(Y[i]| \mu [i], \tau)$

where $\mu [i] = \alpha + \beta * x[i]$

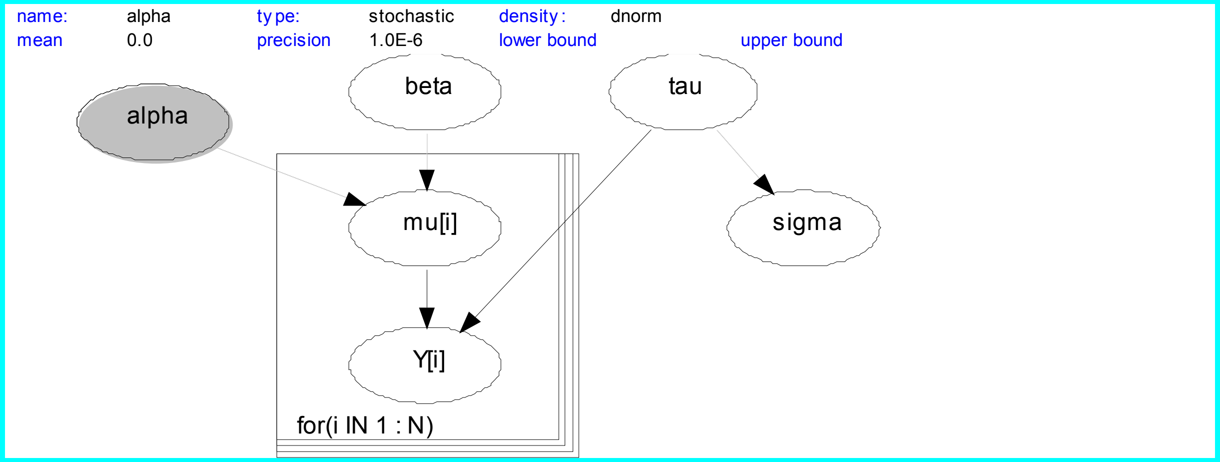
Conditional distributions

- Product of factors
- Subset of factors from the joint probability distribution
- $P(\alpha | \beta, \tau, Y)$

$$= N(\alpha | 0, 1.0E\text{-}3) \prod N(Y[i] | \mu[i], \tau)$$

A graphical view of conditional distributions

- Product of distribution at node labelled by alpha and distributions at nodes that have alpha as a parent



Logical nodes and parents

- If a stochastic node has a logical node as a parent do not regard this as the “true” parent. The “true” parents are the stochastic nodes that are parents of this intermediate logical node

$\mu[i]$ is not a “true” parent of Y[i]

α and β are “true” parents of Y[i]

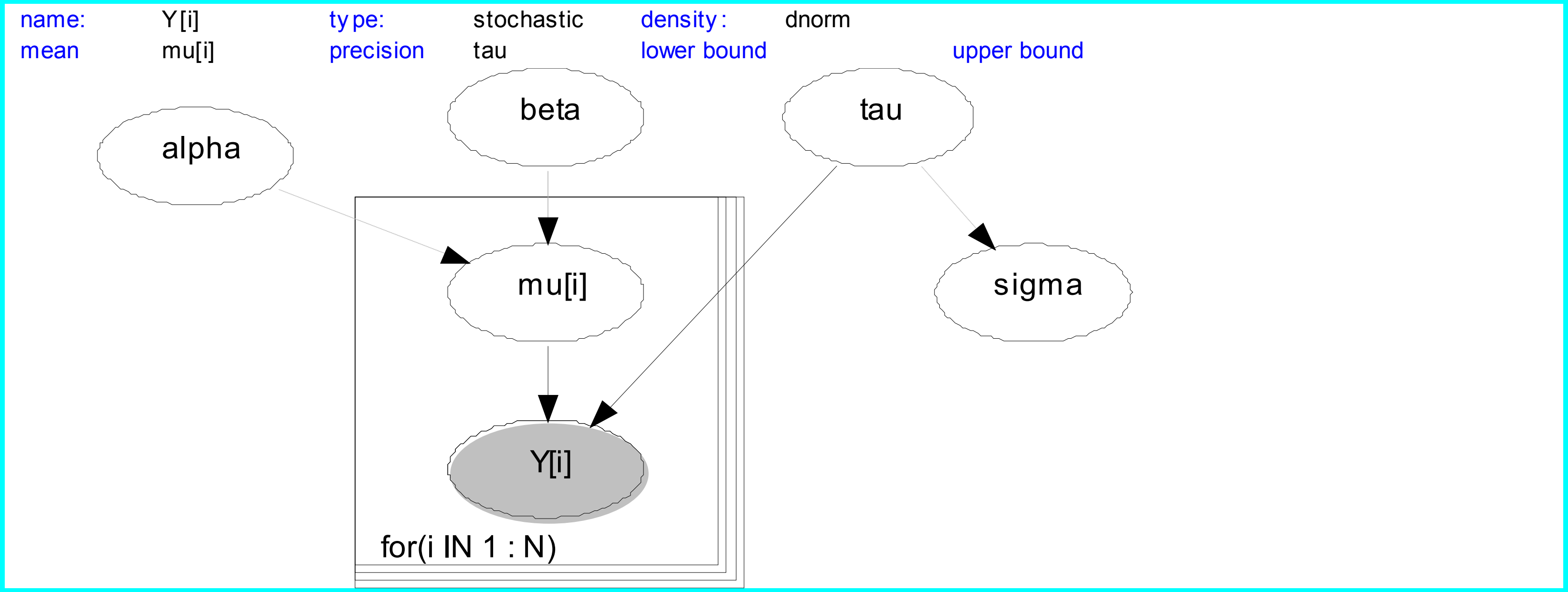
Children of parents

- Graph is specified by giving the parents of each node
- How to find the children of a node – that is nodes with a given parent?
- Go through the whole graph and check if each node has the given node as parent?

Likelihood lists

- The basic algorithm for finding the the children of a given parent can be made much more efficient.
- Store the children of each node as they are found in a list (the likelihood list)
- Can build likelihood lists simultaneously for several nodes

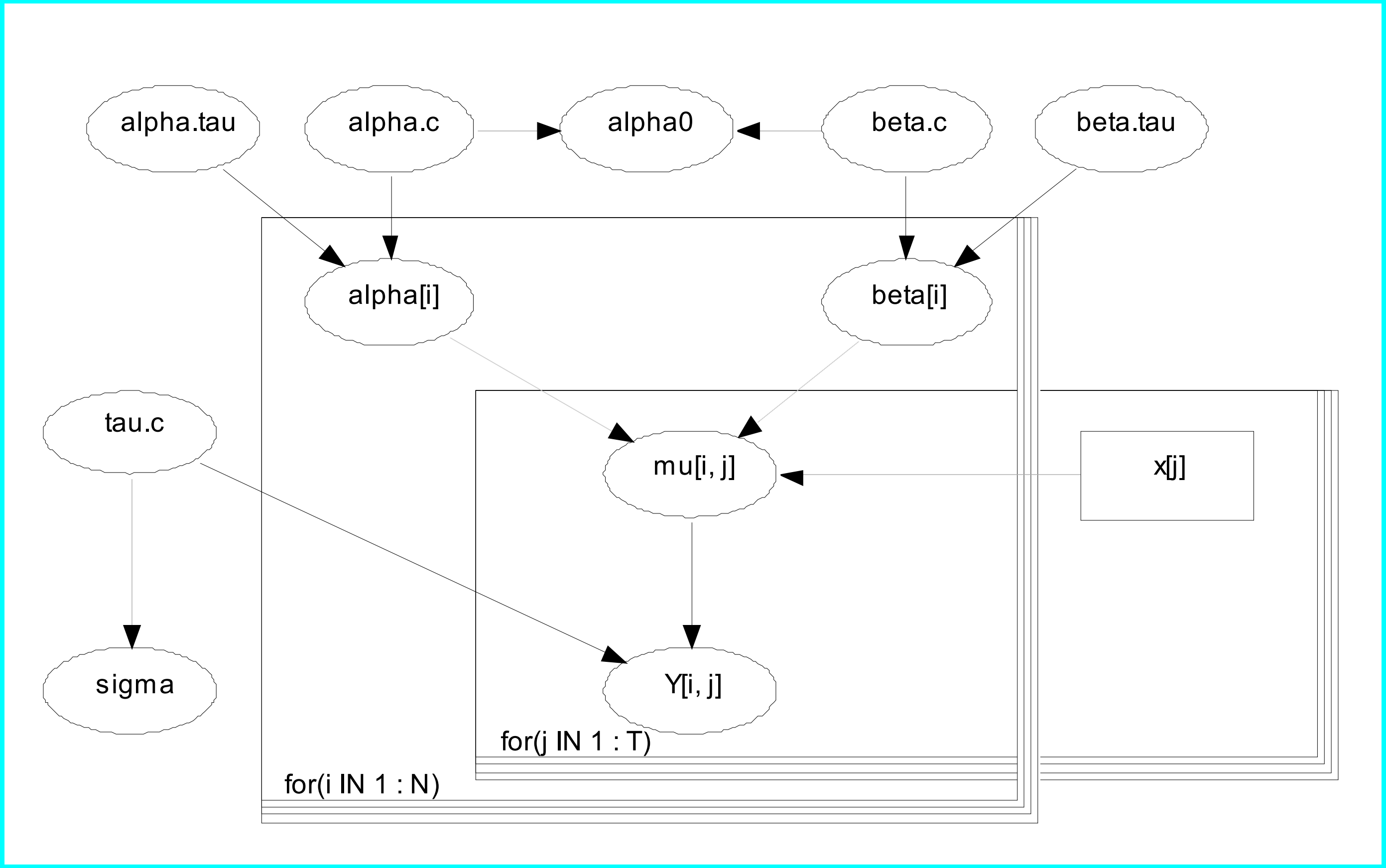
Likelihood lists a simple example



Simple example the details

- Visit each Y[i] node i = 1..5
- Each Y[i] has parents α, β and τ
- Add Y[i] to the likelihood lists for α, β and τ
- Visit α and find its parents (are not any)
- Visit β and find its parents (are not any)
- Visit τ and find its parents (are not any)

A more complex example



Priors and likelihoods

- Call the children of a node the “likelihood”
- Call node itself the “prior”

Y[i, j] likelihood for α [i], β [i] and τ.c

α [i] likelihood for α.c and $\alpha.\tau$

β [i] likelihood for β.c and $\beta.\tau$

Observed and unobserved nodes

- Likelihoods can involve observed nodes, unobserved nodes or a mixture of observed and unobserved nodes.
- In certain cases unobserved nodes need not be included in the likelihood.
- There are simple rules to decide which unobserved nodes must be included in the likelihoods.

Predicting future events

- Can add an extra nodes to graphical model to predict a future event.
- Nodes introduced into the graphical for prediction need not be included in the likelihoods.
- For example in genetic counselling want to advise on the health of a possible future child.

Software

- Graphical models needs software
- WinBUGS is based on graphical models
- WinBUGS is

FUN…