

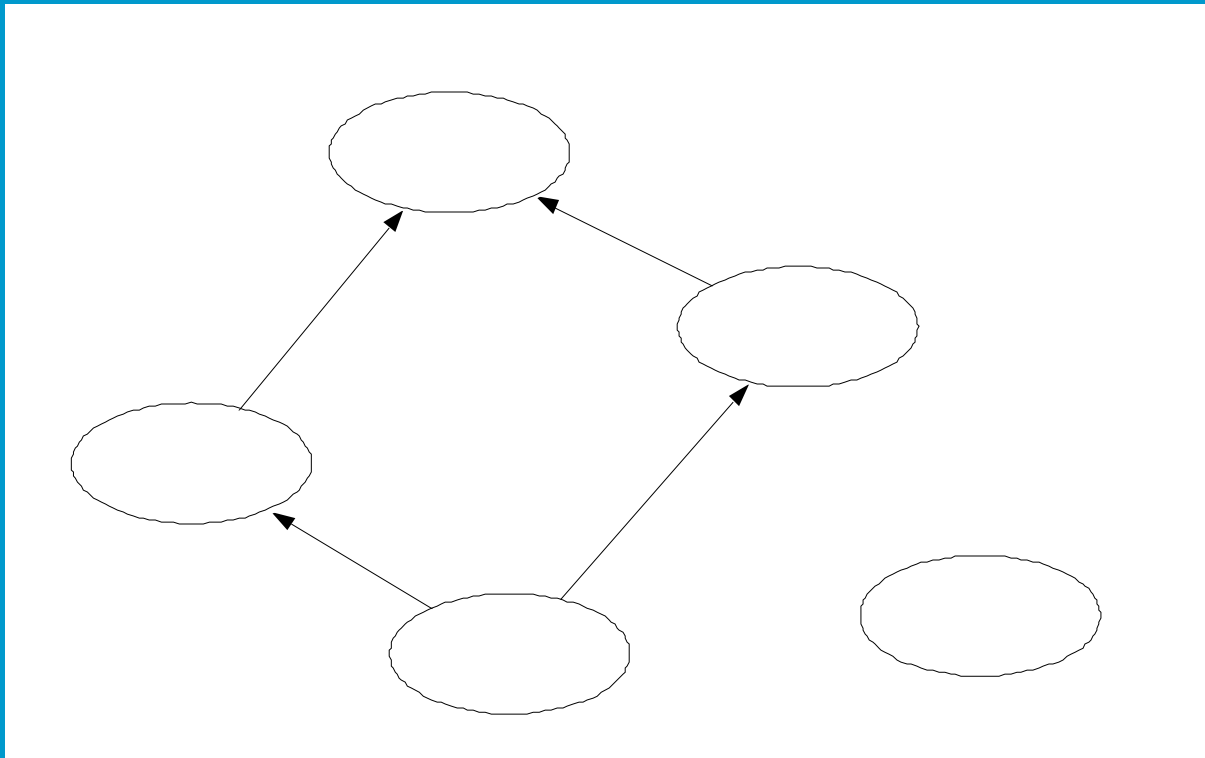
# Graphical Models

## Part One

# Definition of a Graph

- A graph  $G$  is a set of vertices  $V$  and edges  $E$
- $G = \{V, E\}$
- Vertices are also called nodes, we will use the name node.
- Graphs can be represented as algebra or as pictures.

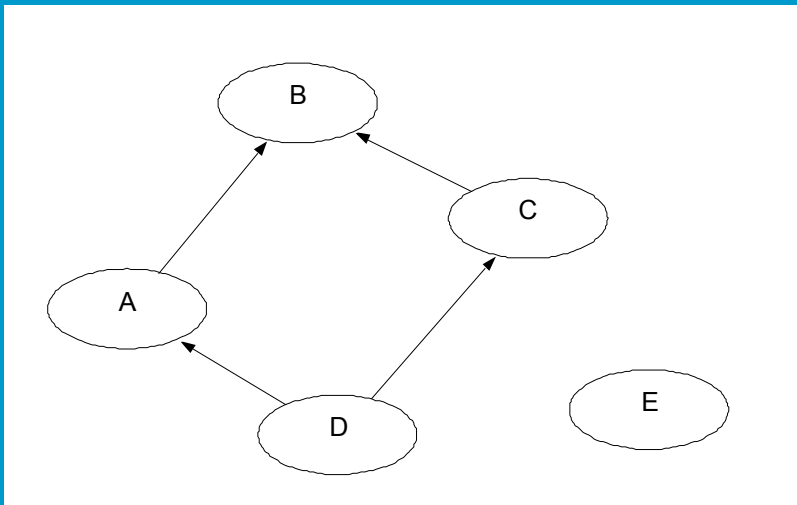
# Example of Pictorial representation of Graph



# Attributing Graphs

- The structure (topology) of a graph can be used to represent a wide range of systems.
- Attaching attributes both to the nodes and the edges of a graph allows complex systems to be modelled.

# Nodes and Quantities

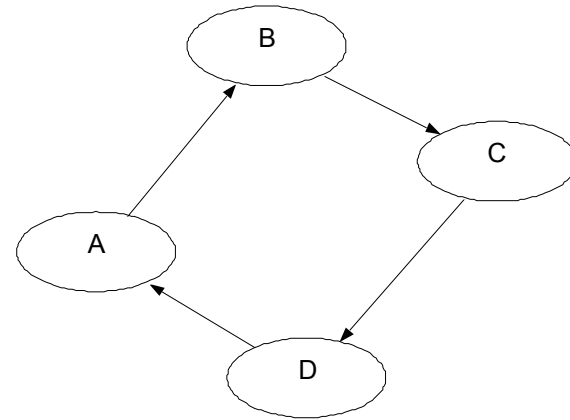
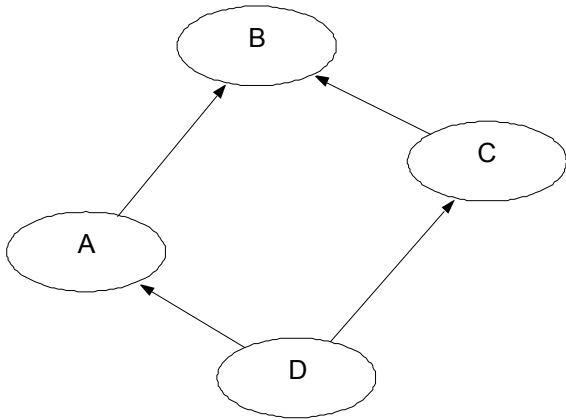


- Nodes in graph have labels.
- Quantities in statistical model have names.

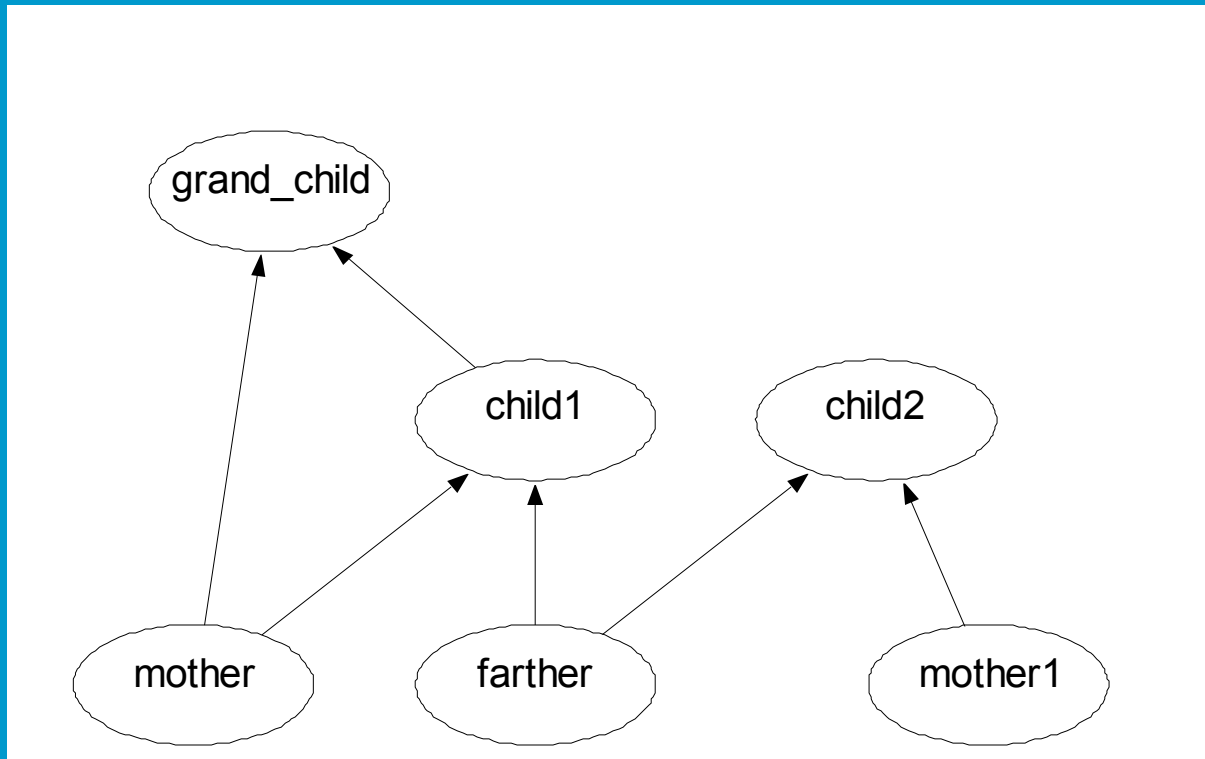
# Edges and Relations

- Edges in a graph can have attributes, different types of edge can be used to describe different types of relations.
- A very important edge attribute is direction, represented by an arrow head.
- Edges can have no arrow head (undirected), a single arrow head (directed) or two opposed arrowheads (bi-directional).

# Directed Graphs and DAGs



# Parents and Children



# Edges can be specified by

- (child1|mother, farther)
- (child2|farther, mother1)
- (grand\_child|mother, child1)

# Or can use a functional notation

- $\text{child1} \sim (\text{mother}, \text{farther})$
- $\text{child2} \sim (\text{farther}, \text{mother1})$
- $\text{grand\_child} \sim (\text{mother}, \text{child1})$

# Probability a new node attribute

- Can associate a probability distribution with each node in the graph.
- The node variable is distributed as the probability distribution.
- The parameters of the distribution are the
- parents of the node.

Example the normal distribution

$$\sqrt{\frac{\tau}{2\pi}} \exp\left(-\frac{\tau}{2}(x - \mu)^2\right); \quad -\infty < x < \infty$$

# Example the gamma distribution

$$\frac{\mu^r x^{r-1} e^{-\mu x}}{\Gamma(r)}; \quad x > 0$$

# Example the binomial distribution

$$\frac{n!}{r!(n-r)!} p^r (1-p)^{n-r}; \quad r = 0, \dots, n$$

# Probabilities can be specified by

- $P(\text{child1} | \text{mother, farther})$
- $P(\text{child2} | \text{farther, mother1})$
- $P(\text{grand\_child} | \text{mother, child1})$

# Or using the functional notation

- $\text{child1} \sim P(\text{mother}, \text{farther})$
- $\text{child2} \sim P(\text{farther}, \text{mother1})$
- $\text{grand\_child} \sim P(\text{mother}, \text{child1})$

# Some Connections

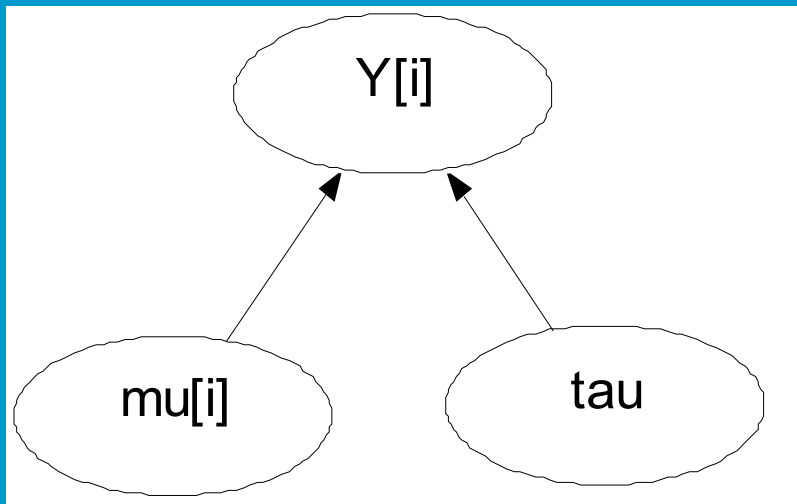
- Joint probability function has a product form: represent as a graph
- BUGS language: a description of a graph
- Both DoodleBUGS and the BUGS language were developed to describe probabilistic models.

# Doodle and the BUGS Language

- Three elements:
  - stochastic relations
  - logical relations
  - repetition

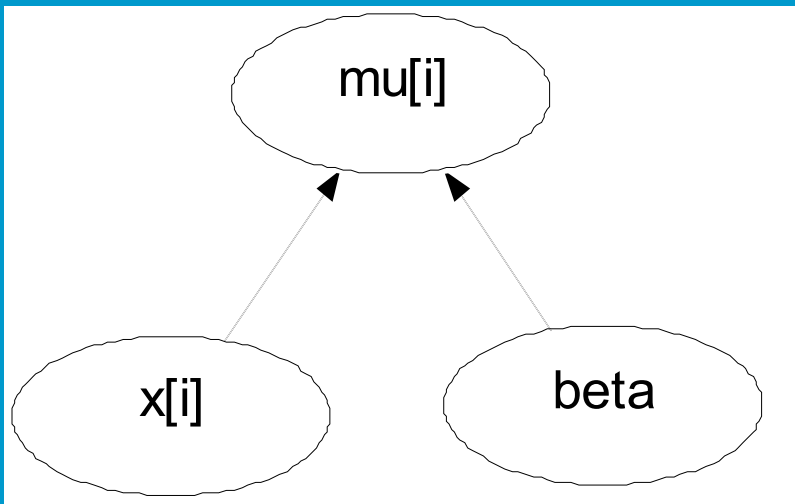
# Stochastic relations

- $Y[i] \sim P(\mu[i], \tau)$



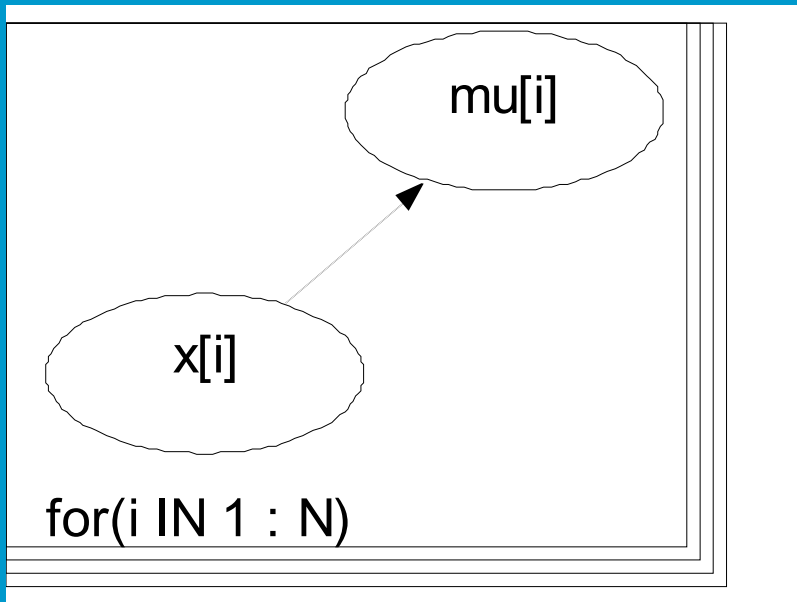
# Logical relations

- $\text{mu}[i] \leftarrow F(\text{x}[i], \text{beta})$

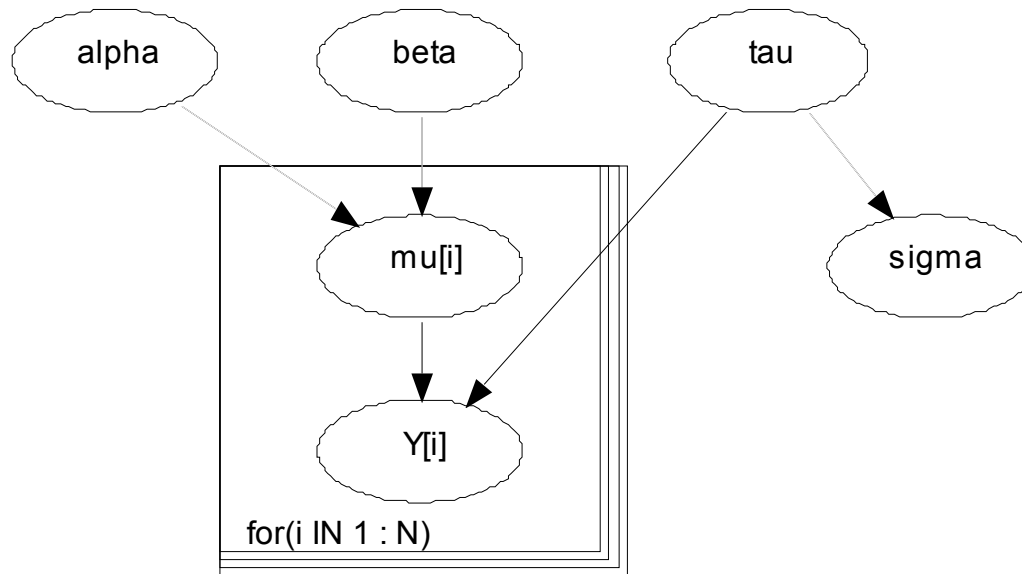


# Repetition

```
for(i in 1:N){  
  mu[i] <- F(beta, x[i])  
}
```



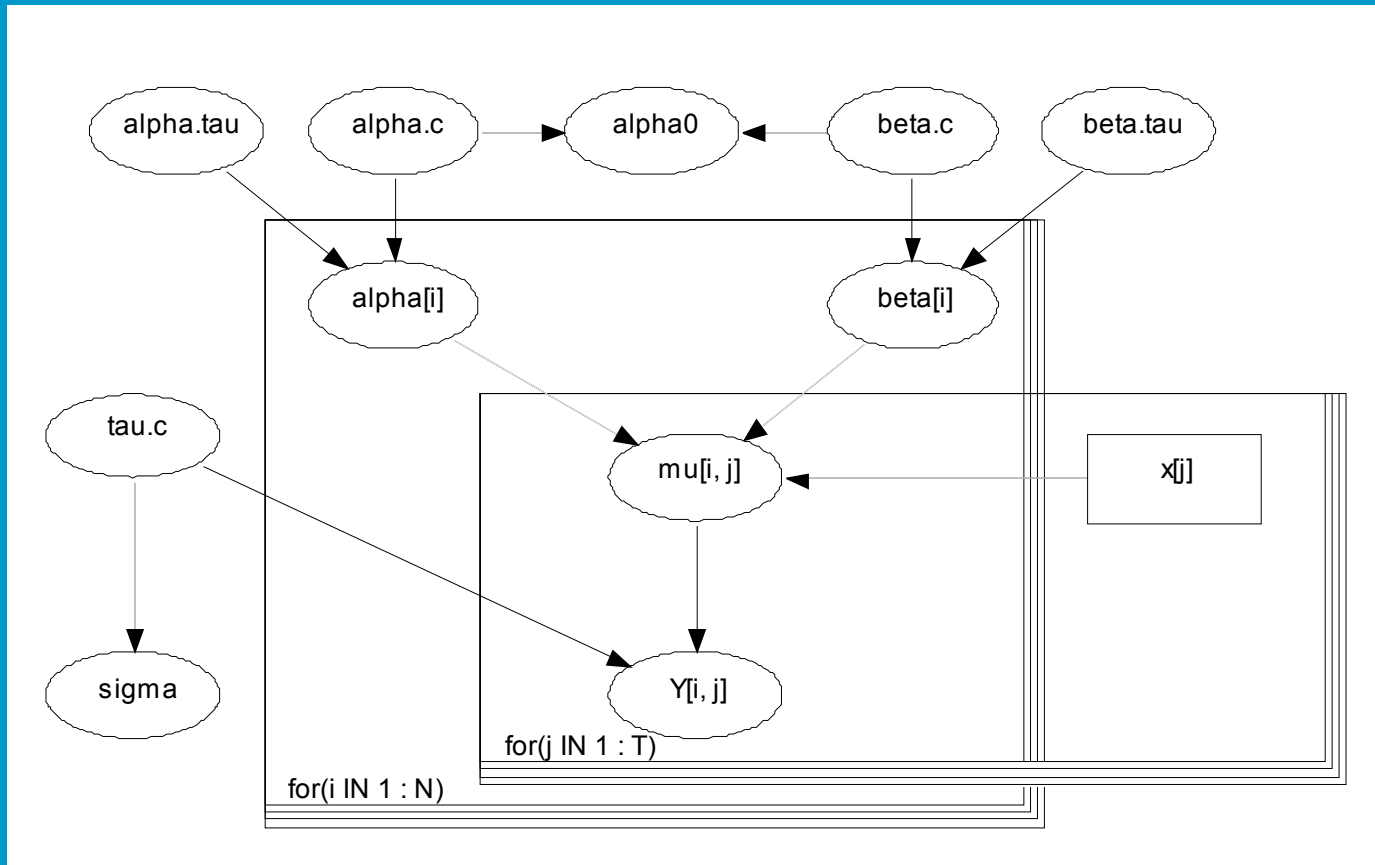
# Line: a simple graphical model



# Line described in BUGS language

```
model{
  for( i in 1 : N ) {
    Y[i] ~ dnorm(mu[i],tau)
    mu[i] <- alpha + beta * (x[i] - xbar)
  }
  tau ~ dgamma(0.001,0.001)
  sigma <- 1 / sqrt(tau)
  alpha ~ dnorm(0.0,1.0E-6)
  beta ~ dnorm(0.0,1.0E-6)
}
```

# Doodle for Rats model



# BUGS language for Rats model

```
model{
  for( i in 1 : N ) {
    for( j in 1 : T ) {
      Y[i , j] ~ dnorm(mu[i , j],tau.c)
      mu[i , j] <- alpha[i] + beta[i] * (x[j] - xbar)
    }
    alpha[i] ~ dnorm(alpha.c, alpha.tau)
    beta[i] ~ dnorm(beta.c, beta.tau)
  }
  tau.c ~ dgamma(0.001,0.001)
  sigma <- 1 / sqrt(tau.c)
  alpha.c ~ dnorm(0.0,1.0E-6)
  alpha.tau ~ dgamma(0.001,0.001)
  beta.c ~ dnorm(0.0,1.0E-6)
  beta.tau ~ dgamma(0.001,0.001)
  alpha0 <- alpha.c - xbar * beta.c
}
```