Hierarchical Models

A Problem

- How big are Swedish frogs?
  - and how does the size vary?
- Variation at several levels
  - between individuals
  - between populations
  - between regions
- Firstly, look at the population and individual levels
  - sample several frogs from different ponds

How to Model

- We want to model the variation between individuals and between ponds
- Treat the county mean as coming from a random distribution
  - e.g. Normal
  - Random effect
- Quantify the amount of variation

The Model

- Individual *i* from pond *j*
- x_{ij} – Size (“Snout Ventral Length)

$$x_{ij} \sim N(\mu_j, \tau)$$

- Common variance
- Means differ between ponds
- Model this as a Normal distribution:

$$\mu_j \sim N(\mu_0, \tau_0)$$

Something Interesting

- For the pond means, we have this:

$$\mu_j \sim N(\mu_0, \tau_0)$$

- If we knew the means, then this would be like a normal inference
  - treating the μ_js as data
- We don't know the means, but we estimate them:

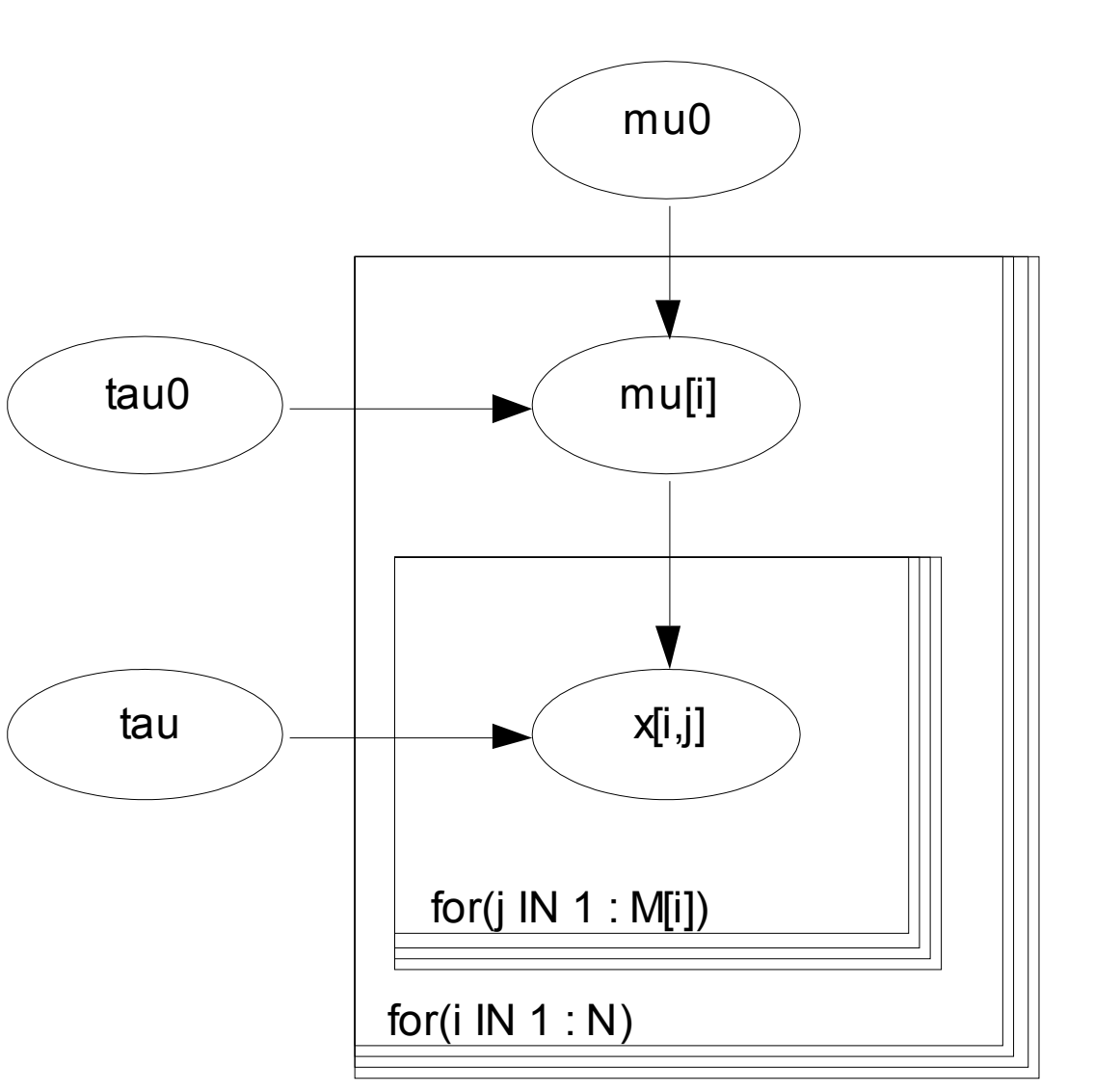
$$x_{ij} \sim N(\mu_j, \tau)$$

- The posterior for μ_0 and τ^2 is then integrated over the uncertainty in the μ_i's

Priors

- For the data (x_{ij}), we can view the distribution of μ_i as the prior for its mean
- So x_{ij} only depends on μ_0 and σ^2 through μ_i
- This makes things easier
  - we can follow the dependencies more easily
- This is a simple *hierarchical model*
- The simple local dependence structure is what makes this modelling easy.

Frogs: The Model



Summarising the Results

- We can summarise the amount of variation that is regional by the proportion of total variation due to the region = ρ:

$$\rho = \frac{1/\tau_0}{1/\tau_0 + 1/\tau}$$

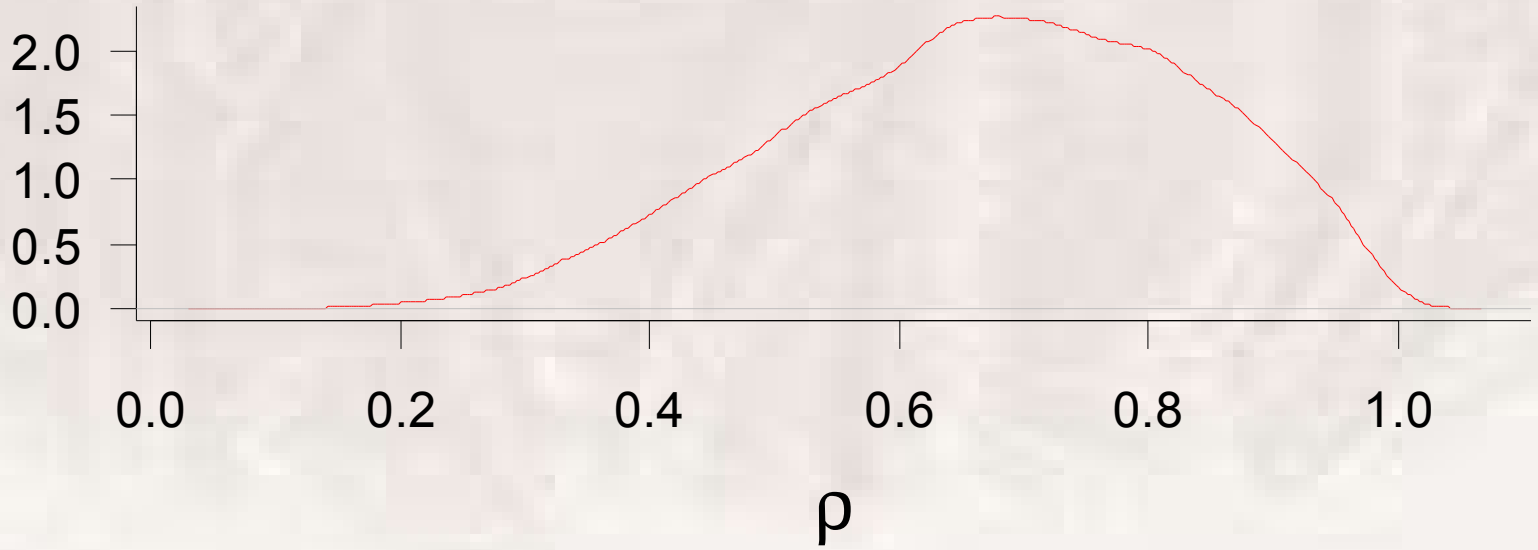
- To calculate this we use each draw from the posterior for τ^2 and τ_0^2 and calculate it from this...

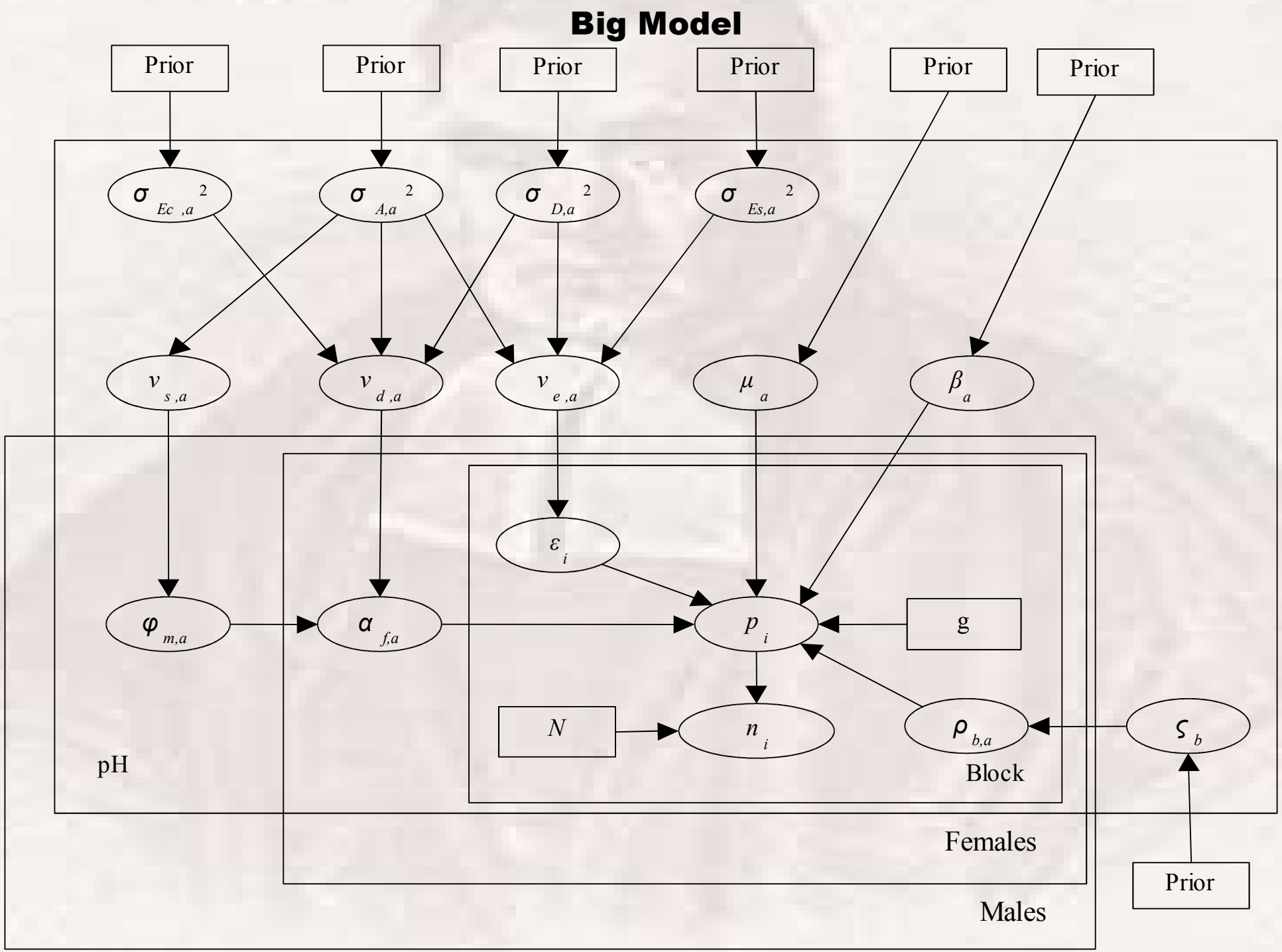
The Estimation

- If we have τ^2 and τ_0^2, we calculate ρ :

τ_0^{-2}	τ^{-2}	ρ
2954	1264	0,7
1594	921	0,63
14444	876	0,94

- And this gives the correct posterior:




Big Model
Prior
Prior
Prior
Prior
Prior
Prior
$\sigma^2_{Ec,a}$
$\sigma^2_{A,a}$
$\sigma^2_{D,a}$
$\sigma^2_{Es,a}$
$v_{s,a}$
$v_{d,a}$
$v_{e,a}$
μ_a
β_a
ε_i
$\varphi_{m,a}$
$\alpha_{f,a}$
p_i
g
N
n_i
$\rho_{b,a}$
ς_b
pH
Block
Females
Males
Prior