

Graphical Models Part Two

Joint probability distribution

- Product of factors

- $P(\alpha, \beta, \tau, Y) = N(\alpha | 0, 1.0E-3) N(\beta | 0, 1.0E-3)$

$$\text{Gamma}(\tau | 1.0E-3, 1.0E-3) \prod N(Y[i] | \mu [i], \tau)$$

where $\mu [i] = \alpha + \beta * x[i]$

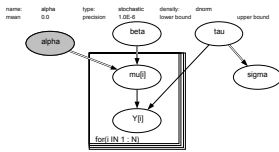
Conditional distributions

- Product of factors
- Subset of factors from the joint probability distribution
- $P(\alpha | \beta, \tau, Y)$

$$= N(\alpha | 0, 1.0E-3) \prod N(Y[i] | \mu [i], \tau)$$

A graphical view of conditional distributions

- Product of distribution at node labelled by alpha and distributions at nodes that have alpha as a parent



Logical nodes and parents

- If a stochastic node has a logical node as a parent do not regard this as the “true” parent. The “true” parents are the stochastic nodes that are parents of this intermediate logical node

$\mu[i]$ is not a “true” parent of $Y[i]$

α and β are “true” parents of $Y[i]$

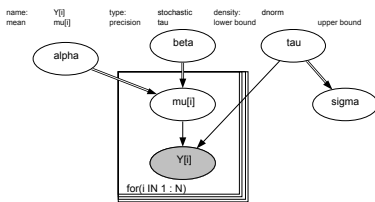
Children of parents

- Graph is specified by giving the parents of each node
- How to find the children of a node – that is nodes with a given parent?
- Go through the whole graph and check if each node has the given node as parent?

Likelihood lists

- The basic algorithm for finding the children of a given parent can be made much more efficient.
- Store the children of each node as they are found in a list (the likelihood list)
- Can build likelihood lists simultaneously for several nodes

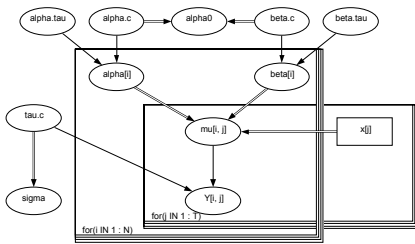
Likelihood lists a simple example



Simple example the details

- Visit each $Y[i]$ node $i = 1..5$
- Each $Y[i]$ has parents α , β and τ
- Add $Y[i]$ to the likelihood lists for α , β and τ
- Visit α and find its parents (are not any)
- Visit β and find its parents (are not any)
- Visit τ and find its parents (are not any)

A more complex example



Priors and likelihoods

- Call the children of a node the “likelihood”
- Call node itself the “prior”

$Y[i, j]$ likelihood for $\alpha[i]$, $\beta[i]$ and $\tau.c$
 $\alpha[i]$ likelihood for $\alpha.c$ and $\alpha.\tau$
 $\beta[i]$ likelihood for $\beta.c$ and $\beta.\tau$

Observed and unobserved nodes

- Likelihoods can involve observed nodes, unobserved nodes or a mixture of observed and unobserved nodes.
- In certain cases unobserved nodes need not be included in the likelihood.
- There are simple rules to decide which unobserved nodes must be included in the likelihoods.

Predicting future events

- Can add an extra nodes to graphical model to predict a future event.
- Nodes introduced into the graphical for prediction need not be included in the likelihoods.
- For example in genetic counselling want to advise on the health of a possible future child.

Software

- Graphical models needs software
- WinBUGS is based on graphical models
- WinBUGS is

FUN...
