



Model Comparison

A General Problem

- We can fit several models to data
- But how do we know which is the best model?
- There are several solutions, here is one which is of some use
 - not always the best!

An Ecological Example

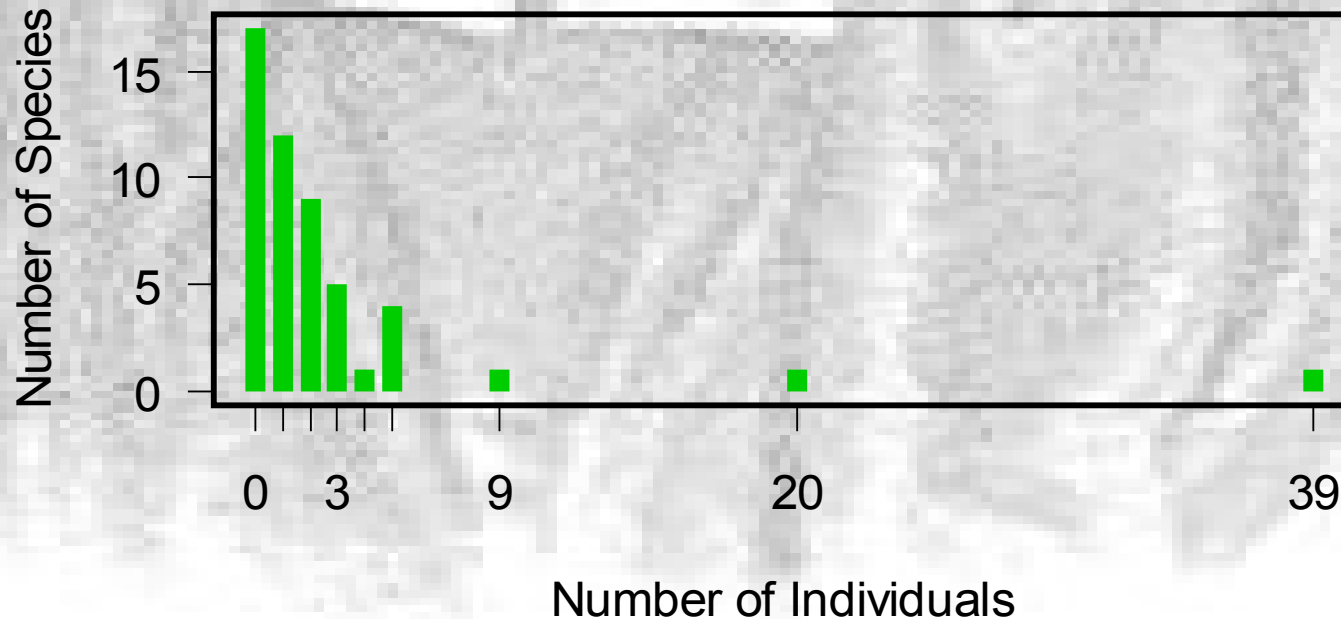
- What is the distribution of abundance?
- A long-standing problem in ecology
- Two (of many) suggestions:
 - gamma
 - lognormal
- The lognormal has some theoretical support, the gamma is more flexible

Some Data

- Mexican Butterflies

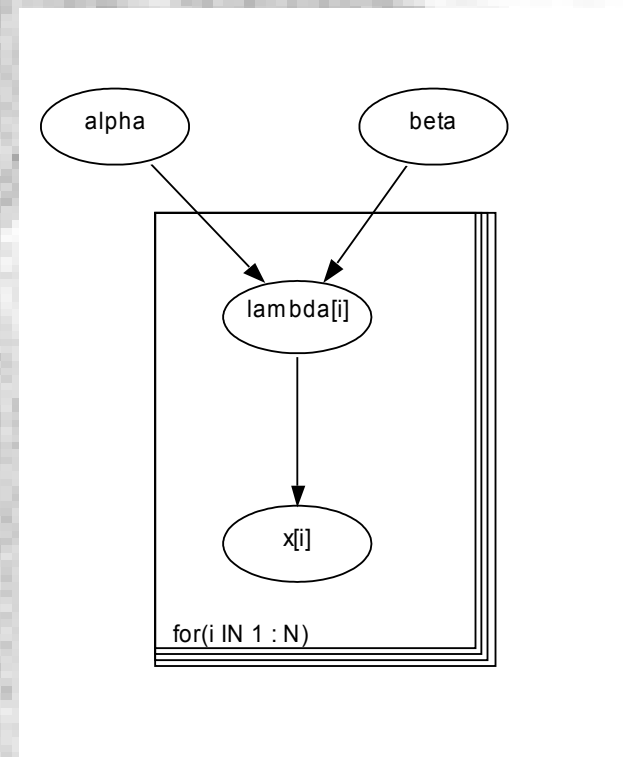
- 39 20 9 5 5 5 5 4 3 3 3 3 3 2 2 2 2 2 2 2
2 2 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0

- Which distribution fits best?



The Models

- For both models, we assume the number of
- individuals follows a Poisson distribution
- – $x[i] \sim \text{dpois}(\text{lambda}[i])$
- 2 models for the mean
 - Gamma
 - $\text{lambda}[i] \sim \text{dgamma}(\text{alpha}, \text{beta})$
 - Log-normal
 - $\log(\text{lambda}[i]) \sim \text{dnorm}(\mu, \tau)$
- Then give priors for alpha, or mu, tau



Comparing Models

- Several approaches in the Bayesian literature
 - Bayes factors
 - rjMCMC
 - DIC
 - SSVS, GVS, CSM,
- In reality there is not true model
 - “All models are wrong, but some are useful”
- Calculating the probability that a model is correct is over-kill
 - instead, take a less formal approach

DIC

- Classical statistics has AIC
 - Akaike's Information Criterion
 - Deviance + $2 \times$ Number of Parameters
- From a similar argument, Bayesians have DIC
 - Deviance Information Criterion
 - $\bar{D} + 2 \times pD$
- Model with the smallest DIC has the highest chance of predicting a replicate data set

What Is DIC?

- We calculate the deviance, D
 - $D = -2 \times \log(P(X | \theta)) = -2 \times \text{likelihood}$
- Take the average over the posterior

$$\bar{D} = - \int 2 \log(P(X|\theta)) d\theta$$

- Or, the average deviance from the MCMC
- Measure of goodness of fit

$$p_D$$

- AIC penalises the deviance with a term involving the number of parameters
 - more complex models get a higher penalty
- DIC penalises with p_D
 - “effective number of parameters”
- Calculated as $D_{\text{bar}} - D_{\text{hat}}$
 - D_{hat} is the deviance at the posterior mean of the parameters

$$\hat{D} = -2 \log(P(X|\bar{\theta}))$$

Using DIC

- If we have several models, we can try and find the one with the lowest DIC measured on the same data!
- If values are close (say, within 5), then the different models are similar makes little difference which one is used
- pD can be interpreted as an estimate of the
- number of parameters
 - almost always positive

Diversity and DIC

- For the butterflies, we get this:
 - Gamma: DIC = 74.16, pD = 28.91
 - log-Normal: DIC = 77.05, pD = 27.36
- The Gamma fits better, but not greatly
- The complexities (pD) are similar
 - about the number of non-zero observations
- Further investigation revealed that neither model is able to fit well
 - most common species too common